IOSR Journal of Electrical and Electronics Engineering (IOSR-JEEE)
e-ISSN: 2278-1676,p-ISSN: 2320-3331, Volume 13, Issue 4 Ver. III (Jul. – Aug. 2018), PP 12-21
www.iosrjournals.org

Cooperative Control for Longitudinal Following and Lateral Lane Changing of Intelligent Cars

Dianbo Ren[1], Hao Wang[2], Yong Zhang[2]
[1](Associate Professor, School of Automotive Engineering, Harbin Institute of Technology at Weihai, China)
[2](Graduate Student, School of Automotive Engineering, Harbin Institute of Technology at Weihai, China)
Corresponding Author: Dianbo Ren

Abstract : *In order to improve the control performance of the intelligent vehicle, by using front and rear wheel active steering technology, in this paper, we study the cooperative control for lane changing and following control of intelligent networked cars. Assuming that the lateral positions of the vehicle in a lane changing transition satisfy five order polynomial constraints, and the longitudinal positions and status information of the vehicle to be followed is obtained by means of inter-vehicle communication. Based on the desired longitudinal and lateral vehicle states, the reference yaw angle for lane changing was generated. Based on the dynamical model of the front and rear wheel steering vehicle, the longitudinal error model of vehicle following and lateral error model of trajectory tracking for lane changing are established. By applying terminal sliding mode technology, the cooperative tracking control law for lane changing and vehicle following was designed. Based on Lyapunov function method, the stability of the control system was obtained. Expected control performance for vehicle following and trajectory tracking for lane changing is verified by the simulation, the asymptotic convergence of longitudinal and lateral position error and yaw angle error is guaranteed.*

Keywords *- cooperative control, intelligent cars, lane changing, terminal sliding mode, vehicle following*

Date of Submission: 13-08-2018 Date of acceptance: 30 -08-2018

I. Introduction

Intelligent vehicle is a comprehensive system integrating the functions of environment perception, planning decision-making and multi-class assistant driving[1]. It is a typical high-tech complex with the technology of computer, modern sensing, information fusion, communication, artificial intelligence and automatic control etc. In recent years, with the development of network communication technology, the intelligent network connected vehicle has gradually become one of the research hotspots in the field of vehicle engineering. Many developed countries have incorporated it into their own intelligent transportation systems [2,3].

Research on intelligent vehicle automatic control mainly includes longitudinal control and lateral control. The vehicle following belongs to the longitudinal control of the vehicle, the purpose of which is to automatically adjust the speed of the controlled vehicle to keep the distance from the preceding vehicle within a set range[4-6]. Since stability is a basic requirement of the control system, the design of the controller in the system needs to be based on the stability analysis of the system. In particular, when the vehicle runs in a platoon, the control law is designed not only to ensure that the motion of each vehicle is stable but also to ensure the group stability of the vehicle platoon. The vehicle lane change belongs to the research field of lateral control for intelligent vehicle, which refers to the process of controlling vehicles from one lane into another lane along the desired trajectory[7-11]. There are many methods for planning the lane change desired trajectory, such as assuming that the vehicle position meets an arc, a sine function, or a polynomial constraint [12]. Because the moving state of the vehicle is also influenced by the longitudinal motion of the front and rear vehicles during the lane change process, when the lane change trajectory tracking control is carried out, the longitudinal and lateral motion state information of the vehicle should be comprehensively considered, and the trajectory tracking error model should be established in the global coordinate system, and the lane change control law should be designed to ensure the stability of the tracking error. Ref.[13] considered the following behavior of vehicles in the process of the lane change and designed the longitudinal and lateral coupling control laws for vehicle lane change. However, for convenience, the research work is based on the vehicle coordinate system, ignoring the correlation between the longitudinal and lateral tracking errors. In addition, a vehicle dynamic model based on front wheel steering is adopted. According to the research result of ref.[14], when the trajectory tracking control is performed on the front wheel steering vehicle, it is not guaranteed that the position tracking error and the yaw angle error simultaneously tend to 0. In Ref. [15], the four-wheel steering dynamic model is used to study the vehicle lane change tracking control, which can simultaneously ensure the asymptotic stability of the position tracking error and the yaw angle error. However, it is assumed that the longitudinal speed is fixed and does not take into account the real-time variation of vehicle longitudinal speed during a lane changing behavior. Refs.[16] and [17] studied the longitudinal control law of vehicle speed during the lane change process according to the longitudinal motion requirements of vehicles. The kinematics model is adopted in this paper, but the dynamic behavior of vehicle motion is not considered. Based on the four-wheel steering vehicle model, ref.[18] studied the lateral control for lane keeping of vehicle without considering the longitudinal following control of the vehicle. Ref.[19] proposed an platoon control algorithm with varying desiring inter-vehicle space, but does not involve the vehicle lateral control.

Based on the global coordinate system, this paper continues to study the cooperative control of vehicle following and lane change trajectory tracking based on the above researches. According to the lateral motion constraint of the vehicle during the lane change process, the lane changing trajectory based on the five-order multinomial is designed, and the longitudinal positions and status information of the vehicle to be followed is obtained by means of inter-vehicle communication. The dynamic models of longitudinal tracking error for vehicle following and the lateral tracking error for lane change are established according to the relationship between the global coordinate system and the vehicle coordinate system. Based on the front and rear wheel active steering dynamics model, the terminal sliding mode control law is designed to ensure the asymptotic stability of the longitudinal and lateral position tracking error and the yaw angle error. The research results are verified by the simulation.

II. Tracking Error Model

2.1 Tracking error definition

Assuming that there are two lanes in the road, lane 1 and lane 2, and the distance between lanes is *d*. The global orthogonal coordinate system is established with a point on lane 1 as the origin, in which the X axis coincides with the center line of lane 1. The direction of travel of the vehicle is positive; the Y axis is perpendicular to the X axis, and the direction from the lane 1 to the lane 2 is positive. The guided vehicle in a motorcade is M_0, and M_i is that the i following vehicle in the platoon, $i = 1,2,\cdots,\mathrm{n}$, in which n is the number of the following vehicles. It is assumed that the displacement, velocity and acceleration information of the guided vehicle can be transmitted to each following vehicle behind it, and the displacement, velocity and acceleration information of the previous vehicle can be transmitted to the next vehicle.

The vehicle spacing error between M_i and M_{i-1} is defined as:

$$\varepsilon_{i,i-1}(t) = X_i(t) - X_{i-1}(t) + \mathrm{L}_i \tag{1}$$

Where L_i denotes the expected distance between the i vehicle and the i-1 vehicle.

The vehicle spacing error between M_0 and M_i is defined as:

$$\varepsilon_{i,0}(t) = X_i(t) - X_0(t) + \sum_{j=1}^{i} \mathrm{L}_j \tag{2}$$

Then from the above equations (1) and (2), we define the longitudinal tracking error of the vehicle M_i as follows.

$$e_{iX}(t) = \varepsilon_{i,i-1}(t) + \eta\varepsilon_{i,0}(t) \tag{3}$$

Where $\eta>0$, is a fixed parameters. The desired lane change trajectory should be tracked along the Y axis when the vehicle M_i needs to change lanes during the following process. The lateral tracking error of the vehicle M_i is defined as

$$e_{iY}(t) = Y_i(t) - Y_{id}(t) \tag{4}$$

Where $Y_{id}(t)$ denotes the lateral desired displacement of the vehicle M_i during the lane change process.

According to ref.[20], the expectation changing trajectory model based on the five-order multinomial can be expressed as

$$Y_{id}(t) = \alpha_5 t^5 + \alpha_4 t^4 + \alpha_3 t^3 + \alpha_2 t^2 + \alpha_1 t + \alpha_0 \tag{5}$$

where α_k is the undetermined coefficient, $k = 0,1,\cdots,5$. Let

$$\mathrm{A} = [\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5]^{\mathrm{T}}$$

So, along this trajectory, the desired velocity and acceleration can be calculated as follows.

$$\begin{cases} \dot{Y}_{id}(t) = 5\alpha_5 t^4 + 4\alpha_4 t^3 + 3\alpha_3 t^2 + 2\alpha_2 t + \alpha_1 \\ \ddot{Y}_{id}(t) = 20\alpha_5 t^3 + 12\alpha_4 t^2 + 6\alpha_3 t^2 + 2\alpha_2 \end{cases} \tag{6}$$

At the beginning and end time of lane change, the displacements of vehicle along the direction of Y axis are $Y_d(t_{on})=0$ and $Y_d(t_{off})=d$, in which the start time is represented by t_{on} and the finish time is t_{off}. The position and the expected speed and acceleration of the vehicle at the beginning and end time are substituted into equations (1) and (2), By solving the linear equations:

$$[0, \dot{Y}_{id}(t_{on}), \ddot{Y}_{id}(t_{on}), \mathrm{d}, \dot{Y}_{id}(t_{off}), \ddot{Y}_{id}(t_{off})]^{\mathrm{T}} = \mathrm{TA} \qquad (7)$$

We get the undetermined coefficient a_k of trajectory model (5), where

$$T = \begin{bmatrix} t_{on}^5 & t_{on}^4 & t_{on}^3 & t_{on}^2 & t_{on} & 1 \\ 5t_{on}^4 & 4t_{on}^3 & 3t_{on}^2 & 2t_{on} & 1 & 0 \\ 20t_{on}^3 & 12t_{on}^2 & 6t_{on} & 2 & 0 & 0 \\ t_{off}^5 & t_{off}^4 & t_{off}^3 & t_{off}^2 & t_{off} & 1 \\ 5t_{off}^4 & 4t_{off}^3 & 3t_{off}^2 & 2t_{off} & 1 & 0 \\ 20t_{off}^3 & 12t_{off}^2 & 6t_{off} & 2 & 0 & 0 \end{bmatrix}$$

The direction of vehicle longitudinal velocity should be consistent with the tangential direction of lane changing trajectory, so the desired yaw angle ψ_{id} of the vehicle M_i should be equal to the angle between the direction of the trajectory and the direction of the lane, we have:

$$\psi_{id}(t) = \operatorname{atan}\frac{\dot{Y}_{id}(t)}{\dot{X}_{id}(t)} \qquad (8)$$

We define the yaw angle tracking error as following:

$$e_{i\psi}(t) = \psi_i(t) - \psi_{id}(t) \qquad (9)$$

Where ψ_i is the yaw angle of the controlled vehicle.

2.2 Error dynamic model

In the body coordinate system, assume that the longitudinal displacement of the vehicle is x, the lateral displacement of the vehicle is y, and the angle between the longitudinal axis of the vehicle and the X axis of the global coordinate system is ψ, as shown in figure 1, we have:

$$\begin{cases} v_1 = \dot{x}\sin\psi \\ v_2 = \dot{x}\cos\psi \\ v_3 = \dot{y}\sin\psi \\ v_4 = \dot{y}\cos\psi \end{cases} \qquad (10)$$

Where $\dot{x}$ denotes longitudinal velocity and $\dot{y}$ denotes lateral velocity.

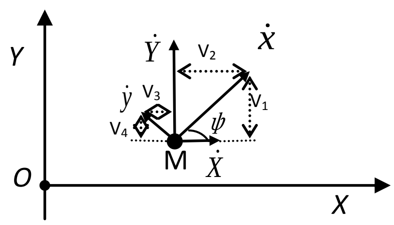


Fig.1: Position relationship between body coordinate system and global coordinate system

From Fig.1, the speed along the axis of the vehicle in the global coordinate system can be expressed as:

$$\begin{cases} \dot{X}_i = \dot{x}_i\cos\psi_i - \dot{y}_i\sin\psi_i \\ \dot{Y}_i = \dot{x}_i\sin\psi_i + \dot{y}_i\cos\psi_i \end{cases} \qquad (11)$$

From the above equations (1) and (2), we get:

$$\dot{\varepsilon}_{i,i-1}(t) = \dot{x}_i\cos\psi_i - \dot{y}_i\sin\psi_i - \dot{X}_{i-1}(t) \qquad (12)$$

$$\varepsilon_{i,0}(t) = \dot{x}_i\cos\psi_i - \dot{y}_i\sin\psi_i - \dot{X}_0(t) \qquad (13)$$

Then from the above equations (3), (4) and (9), we get

$$\begin{cases} \dot{e}_{i\psi} = \dot{\psi}_i - \dot{\psi}_{id} \\ \dot{e}_{iX} = \dot{x}_i\cos\psi_i - \dot{y}_i\sin\psi_i - \dot{X}_{i-1}(t) \\ \qquad + \eta[\dot{x}_i\cos\psi_i - \dot{y}_i\sin\psi_i - \dot{X}_0(t)] \\ \dot{e}_{iY} = \dot{x}_i\sin\psi_i + \dot{y}_i\cos\psi_i - \dot{Y}_{id} \end{cases} \qquad (14)$$

Considering the equation (11), the acceleration along the axis can be expressed

$$\begin{cases} \ddot{X}_i = \ddot{x}_i \cos\psi_i - \ddot{y}_i \sin\psi_i + w_{i1} \\ \ddot{Y}_i = \ddot{x}_i \sin\psi_i + \ddot{y}_i \cos\psi_i + w_{i2} \end{cases} \tag{15}$$

Where

$$w_{i1} = -\dot{x}_i\dot{\psi}_i \sin\psi_i - \dot{y}_i\dot{\psi}_i \cos\psi_i \tag{16}$$

$$w_{i2} = \dot{x}_i\dot{\psi}_i \cos\psi_i - \dot{y}_i\dot{\psi}_i \sin\psi_i \tag{17}$$

From the equation (14), we have:

$$\begin{cases} \ddot{e}_{i\psi} = \ddot{\psi}_i - \ddot{\psi}_{id} \\ \ddot{e}_{iX} = (1+\eta)\left[\ddot{x}_i \cos\psi_i - \ddot{y}_i \sin\psi_i + w_{i1}\right] \\ \qquad - \ddot{X}_{i-1} - \eta\ddot{X}_0 \\ \ddot{e}_{iY} = \ddot{x}_i \sin\psi_i + \ddot{y}_i \cos\psi_i + w_{i2} - \ddot{Y}_{id} \end{cases} \tag{18}$$

III. Control System Design

3.1 Vehicle dynamics model

The vehicle dynamics model adopted in this paper stems from the ideal model proposed by Ackermann, without considering the influence of road slope and roll motion. From Automotive theory, a longitudinal and lateral dynamics model is given by

$$\ddot{x}_i = \frac{1}{\delta m_i}\left(F_i - m_i g f_R + m_i \dot{y}_i\dot{\psi}_i - C_{iA}\dot{x}_i^{\,2}\right) \tag{19}$$

$$\ddot{y}_i = -\frac{2(C_{if}+C_{ir})}{m_i\dot{x}_i}\dot{y}_i - \left[\dot{x}_i + \frac{2(C_{if}l_{if} - C_{ir}l_{ir})}{m\dot{x}_i}\right]\dot{\psi}_i$$

$$+ \frac{2C_f}{m}\delta_f + \frac{2C_r}{m}\delta_r \tag{20}$$

$$\ddot{\psi}_i = -\frac{2(C_{if}l_{if}^2 + C_{ir}l_{ir}^2)}{I_{iz}\dot{x}_i}\dot{\psi}_i - \frac{2(C_{if}l_{if} - C_{ir}l_{ir})}{I_{iz}\dot{x}_i}\dot{y}_i$$

$$+ \frac{2C_{if}l_{if}}{I_{iz}}\delta_{if} - \frac{2C_{ir}l_{ir}}{I_{iz}}\delta_{ir} \tag{21}$$

Where m_i is the total mass of vehicle M_i, δ is the correction coefficient of rotating mass, I_{iz} is the total inertia about vertical axis of vehicle M_i, F_i is the driving or braking force on the wheel, f_R is rolling resistance coefficient, C_{iA} is an air resistance coefficient, l_{if} and l_{ir} are the distance of front and rear axle from center of gravity of vehicle, respectively, C_{if} and C_{ir} are the front and rear tire cornering stiffness, respectively, δ_{if} and δ_{ir} are the front and rear wheel steering angle.

Let

$$\begin{cases} u_i = \dfrac{F_i}{\delta m_i} - \dfrac{g f_R}{\delta} \\ a_{i1} = \dfrac{1}{\delta} \\ a_{i2} = -\dfrac{C_{iA}}{\delta m_i} \end{cases} \tag{22}$$

$$\begin{cases} b_{i1} = -\dfrac{2(C_{if} + C_{ir})}{m_i\dot{x}_i} \\ b_{i2} = -\dot{x}_i - \dfrac{2(C_{if}l_{if} - C_{ir}l_{ir})}{m_i\dot{x}_i} \end{cases} \tag{23}$$

$$\begin{cases} c_{i1} = -\dfrac{2(C_{if}l_{if}^2 + C_{ir}l_{ir}^2)}{I_{iz}\dot{x}_i} \\ c_{i2} = -\dfrac{2(C_{if}l_{if} - C_{ir}l_{ir})}{I_{iz}\dot{x}_i} \end{cases} \tag{24}$$

$$\begin{cases} \upsilon_{i1} = \dfrac{2C_{if}}{m_i}\delta_{if} + \dfrac{2C_{ir}}{m_i}\delta_{ir} \\ \upsilon_{i2} = \dfrac{2C_{if}l_{if}}{I_{iz}}\delta_{if} - \dfrac{2C_{ir}l_{ir}}{I_{iz}}\delta_{ir} \end{cases} \tag{25}$$

The equations (19)-(21) can be rewritten as

$$\begin{cases} \ddot{x}_i = a_{i1}\dot{y}_i\dot{\psi}_i + a_{i2}\dot{x}_i^2 + u_i \\ \ddot{y}_i = b_{i1}\dot{y}_i + b_{i2}\dot{\psi}_i + \upsilon_{i1} \\ \ddot{\psi}_i = c_{i1}\dot{\psi}_i + c_{i2}\dot{y}_i + \upsilon_{i2} \end{cases} \tag{26}$$

3.2 Tracking control law

Applying sliding mode technology, we design the switch function as

$$\begin{cases} S_{iX} = \dot{e}_{iX} + \rho_{i1}e_{iX} + \varphi_{i1}e_{iX}^{k_{i1}/l_{i1}} \\ S_{iY} = \dot{e}_{iY} + \rho_{i2}e_{iY} + \varphi_{i2}e_{iY}^{k_{i2}/l_{i2}} \\ S_{i\psi} = \dot{e}_{i\psi} + \rho_{i3}e_{i\psi} + \varphi_{i3}e_{i\psi}^{k_{i3}/l_{i3}} \end{cases} \tag{27}$$

Where $\rho_{ij}>0, \psi_{ij}>0, k_{ij}$ and l_{ij} are positive odd number, $l_{ij}>k_{ij}$, $j=1,2,3$.

Calculating the derivative of equation (27), from equation (15), we obtain

$$\begin{cases} \dot{S}_{iX} = (1+\eta)[\ddot{x}_i\cos\psi_i - \ddot{y}_i\sin\psi_i + w_{i1}] \\ \qquad - \ddot{X}_{i-1} - \eta\ddot{X}_0 + \rho_{i1}\dot{e}_{iX} + \dfrac{\varphi_{i1}k_{i1}}{l_{i1}}e_{iX}^{k_{i1}/l_{i1}-1}\dot{e}_{iX} \\ \dot{S}_{iY} = \ddot{x}_i\sin\psi_i + \ddot{y}_i\cos\psi_i + w_{i2} - \ddot{Y}_{id} \\ \qquad + \rho_{i2}\dot{e}_{iY} + \dfrac{\varphi_{i2}k_{i2}}{l_{i2}}e_{Y}^{k_{i2}/l_{i2}-1}\dot{e}_{iY} \\ \dot{S}_{i\psi} = \ddot{\psi}_i - \ddot{\psi}_{id} + \rho_{i3}\dot{e}_{i\psi} + \dfrac{\varphi_{i3}k_{i3}}{l_{i3}}e_{Y}^{k_3/l_3-1}\dot{e}_{i\psi} \end{cases} \tag{28}$$

Let $\dot{S}_{iX} = 0, \dot{S}_{iY} = 0, \dot{S}_{iY} = 0$, then from the above equations (27) and (28), we get

$$\begin{cases} \ddot{x}_i\cos\psi_i - \ddot{y}_i\sin\psi_i - \Delta_{i1} = 0 \\ \ddot{x}_i\sin\psi_i + \ddot{y}_i\cos\psi_i - \Delta_{i2} = 0 \\ \ddot{\psi}_i - \Delta_{i3} = 0 \end{cases} \tag{29}$$

Where

$$\begin{cases} \Delta_{i1} = -w_{i1} + (\ddot{X}_{i-1} + \eta\ddot{X}_0 - \rho_{i1}\dot{e}_{iX} \\ \qquad - \dfrac{\varphi_{i1}k_{i1}}{l_{i1}}e_{iX}^{k_{i1}/l_{i1}-1}\dot{e}_{iX})/(1+\eta) \\ \Delta_{i2} = -w_{i2} + \ddot{Y}_{id} - \rho_{i2}\dot{e}_{Y} - \dfrac{\varphi_{i2}k_{i2}}{l_{i2}}e_{iY}^{k_2/l_2-1}\dot{e}_{iY} \\ \Delta_{i3} = \ddot{\psi}_{id} - \rho_{i3}\dot{e}_{i\psi} - \dfrac{\varphi_{i3}k_{i3}}{l_{i3}}e_{i\psi}^{k_{i3}/l_{i3}-1}\dot{e}_{i\psi} \end{cases} \tag{30}$$

From the equation (29), we get

$$\begin{cases} \ddot{x}_i = \Delta_{i1}\cos\psi_i + \Delta_{i2}\sin\psi_i \\ \ddot{y}_i = \Delta_{i1}\sin\psi_i - \Delta_{i2}\cos\psi_i \\ \ddot{\psi}_i = \Delta_{i3} \end{cases} \tag{31}$$

From the vehicle dynamics model (26), we get the equivalent control law as

$$\begin{cases} u_{iequ} = \Delta_{i1}\cos\psi_i + \Delta_{i2}\sin\psi_i - a_{i1}\dot{y}_i\dot{\psi}_i - a_{i2}\dot{x}_i^2 \\ v_{i1equ} = \Delta_{i1}\sin\psi_i - \Delta_{i2}\cos\psi_i - b_{i1}\dot{y}_i - b_{i2}\dot{\psi}_i \\ v_{i2equ} = \Delta_{i3} - c_{i1}\dot{\psi}_i - c_{i2}\dot{y}_i \end{cases} \tag{32}$$

Let $\mathrm{U}_{iequ} = [u_{iequ}, v_{i1equ}, v_{i2equ}]^{\mathrm{T}}$.

Adopt the form of approaching law as

$$\dot{\mathrm{S}}_i = -\boldsymbol{\lambda}_i \mathrm{S}_i \tag{33}$$

Where $\mathrm{S}_i = [S_{iX}, S_{iY}, S_{i\psi}]^{\mathrm{T}}$, $\boldsymbol{\lambda}_i = \begin{bmatrix} \lambda_{i1} & & \\ & \lambda_{i2} & \\ & & \lambda_{i3} \end{bmatrix}$, the positive definite switching function parameter matrix.

Design the total control law as

$$\mathrm{U}_i = \mathrm{U}_{iequ} + \mathrm{U}_{iN} \tag{34}$$

Where $\mathrm{U}_i = [u_i, v_{i1}, v_{i2}]^{\mathrm{T}}$, $\mathrm{U_N}$ denotes the sliding mode feedback control, and $\mathrm{U}_N = [u_N, v_{1N}, v_{2N}]^{\mathrm{T}}$.

From equations (26), (28), (32), (33) and (34), we obtain

$$\mathrm{U}_{iN} = -\boldsymbol{\lambda}_i \mathrm{S}_i \tag{35}$$

Then according to the equations (22) and (25), we get control law as

$$\begin{cases} F_i = \delta m_i u_i + m_i g f_R \\ \delta_{if} = (b_{i12}v_{i2} - b_{i22}v_{i1}) / (b_{i12}b_{i21} - b_{i11}b_{i22}) \\ \delta_{ir} = (b_{i11}v_{i2} - b_{i21}v_{i1}) / (b_{i11}b_{i22} - b_{i12}b_{i21}) \end{cases} \tag{36}$$

Where

$$b_{i11} = \frac{2C_{if}}{m_i},\ b_{i12} = \frac{2C_{ir}}{m_i},\ b_{i21} = \frac{2C_{if}l_{if}}{I_{iz}},\ b_{i22} = -\frac{2C_{ir}l_{ir}}{I_{iz}}.$$

3.3 System stability

Define a Lyapunov function as $V_i = \frac{1}{2}(S_{i\psi}^2 + S_{iX}^2 + S_{iY}^2)$, calculating the derivative of equation (22), from equations (18), (19), (20) and (36), we obtain

$$V_i = S_{i\psi}\dot{S}_{i\psi} + S_{iX}\dot{S}_{iX} + S_{iY}\dot{S}_{iY} = -(\lambda_{i1}S_{i\psi}^2 + \lambda_{i2}S_{iX}^2 + \lambda_{i3}S_{iY}^2) \le 0 \tag{37}$$

Only when $S_{i\psi}$, S_{iX} and S_{iY} are zero, we have V_i=0, so $S_{iX}\to 0$, $S_{iY}\to 0$, $S_{i\psi}\to 0$ as $t\to\infty$. That shows the sliding mode is asymptotically reachable.

According to the switching function equation (27), from $\mathrm{S}_i = 0$, we have

$$\begin{cases} \dot{e}_{iX} + \rho_{i1}e_{iX} + \varphi_{i1}e_{iX}^{k_{i1}/l_{i1}} = 0 \\ \dot{e}_{iY} + \rho_{i2}e_{iY} + \varphi_{i2}e_{iY}^{k_{i2}/l_{i2}} = 0 \\ \dot{e}_{i\psi} + \rho_{i3}e_{i\psi} + \varphi_{i3}e_{i\psi}^{k_{i3}/l_{i3}} = 0 \end{cases} \tag{38}$$

Assume that the time when the system reaches the sliding mode is T_j, and $e_j(T_j)\neq 0$, the time for $e_j(T_j)$ to e_j=0 is τ_j, where $\mathbf{T}_i = [T_{iX}, T_{iY}, T_{i\psi}]^{\mathrm{T}}$, $\mathrm{e}_i = [e_{iX}, e_{iY}, e_{i\psi}]^{\mathrm{T}}$, $\boldsymbol{\tau}_i = [\tau_{iX}, \tau_{iY}, \tau_{i\psi}]^{\mathrm{T}}$.

Because $\rho_{ij}>0$, $\psi_{ij}>0$, k_{ij} and l_{ij} are positive odd numbers, $l_{ij}>k_{ij}$, j=1,2,3. Solving differential equation (38), we get

$$\begin{cases} \tau_{iX} = \dfrac{l_{i1}}{(l_{i1} - k_{i1})\rho_{i1}} \ln \dfrac{\varphi_{i1} + \rho_{i1} e_{iX} (T_{iX})^{(l_{i1}-k_{i1})/k_{i1}}}{\varphi_{i1}} \\ \tau_{iY} = \dfrac{l_{i2}}{(l_{i2} - k_{i2})\rho_{i2}} \ln \dfrac{\varphi_{i2} + \rho_{2} e_{iY} (T_{iY})^{(l_{i2}-k_{i2})/k_{i2}}}{\varphi_{i2}} \\ \tau_{i\psi} = \dfrac{l_{i3}}{(l_{i3} - k_{i3})\rho_{i3}} \ln \dfrac{\varphi_{i3} + \rho_{3} e_{i\psi} (T_{i\psi})^{(l_{i3}-k_{i3})/k_{i3}}}{\varphi_{i3}} \end{cases} \tag{39}$$

From equation (39), the system (18) reaches the equilibrium point within a finite time along the sliding mode, it shows that the convergence speed of the system is fast.

According to equation (3), from e_{iX}=0, $e_{i-1,X}$=0, we get

$$\varepsilon_{i,i-1}(t) + \eta\varepsilon_{i,0}(t) = 0 \tag{40}$$

$$\varepsilon_{i-1,i-2}(t) + \eta\varepsilon_{i-1,0}(t) = 0 \tag{41}$$

By the equations (40) and (41), the vehicle spacing error of the adjacent vehicles of the platoon is satisfied as follows.

$$(1 + \eta)\ \varepsilon_{i,i-1}(t) - \varepsilon_{i-1,i-2}(t) = 0 \tag{42}$$

Because $\eta > 0$, further has

$$|\ \varepsilon_{i,i-1}(t)\ | = \frac{1}{1+\eta}\ |\ \varepsilon_{i-1,i-2}(t)\ | \le |\ \varepsilon_{i-1,i-2}(t)\ | \tag{43}$$

Equation (43) shows that when the speed of the guided vehicle changes, the spacing error of adjacent vehicles of the platoon will not increase one after another.

IV. Simulation results

Suppose the platoon consists of five vehicles, one guiding vehicle and four following vehicles. In simulation, correction coefficient of rotating mass δ=1.1, the air resistance coefficient f_R=0.02, and the moment of inertia C_A=0.4, other parameters of the following vehicle are selected as Table 1.

Table 1- Parameters of Vehicle

Parm / S/N	m (kg)	I_z (kgm^2)	l_f (m)	l_r (m)	C_f (kN/rad)	C_r (kN/rad)
1	2100	3150	1.33	1.26	70	80
2	1800	3050	1.30	1.20	58	68
3	1850	2920	1.20	1.30	65	75
4	1900	3120	1.30	1.40	75	85

In simulation, assume that the expected vehicle spacing, L_i=12m, i=1,2,3,4, the initial vehicle spacing error:$\varepsilon_1(0)$=1m, $\varepsilon_2(0)$=0.75m, $\varepsilon_3(0)$=0.5m, $\varepsilon_4(0)$=0.25m, the initial longitudinal velocity:$V_{X0}(0)$=20m/s, $V_{X1}(0)$=19.5m/s, $V_{X2}(0)$=19m/s, $V_{X3}(0)$=18.5m/s, $V_{X4}(0)$=18m/s, initial longitudinal displacement: $X_0(0)$=80m, $X_1(0)$=69m, $X_2(0)$=57.75m, $X_3(0)$=46.25m, $X_4(0)$=34.5m.

The acceleration of the guiding vehicle is as follows.

$$a_0 = \begin{cases} 0 & t < 4 \\ -0.25(t-4) & 4 \le t < 7 \\ -0.75 & 7 \le t < 10 \\ 0.25(t-10) - 0.75 & 10 \le t < 16 \\ 0.75 & 16 \le t < 19 \\ 0.25(19-t) + 0.75 & 19 \le t < 22 \\ 0 & 22 \le t \le 30 \end{cases} \tag{44}$$

Assume that the lane centerline spacing d=3m, the vehicle M_3 changes lane during the following process, the start time of changing lane t_{on}=10s, the end time t_{off}=13.5s. In the global coordinate system, the desired state of the vehicle at the beginning time: $\dot{Y}_{id}(10) = 0.1$ m/s, $\ddot{Y}_{id}(10) = 0.01$ m/s^2, the desired state of the vehicle at the end time: $\dot{Y}_{id}(13.5) = 0$, $\ddot{Y}_{id}(13.5) = 0$. From equation (7), we get

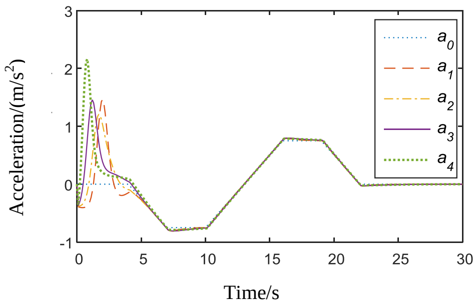
$A = [0.0321, -0.2796, 0.6475, 0, 0.1, 0]^T$

Therefore, the desired lane change trajectory along the *Y* axis can be expressed as

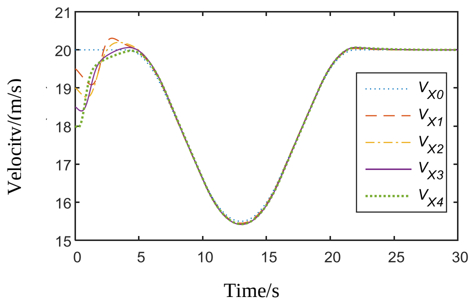
$Y_{id}(t) = 0.0321t^5 - 0.2796t^4 + 0.6475t^3 + 0.1t$

At the start time of vehicle M_3 changes lane, assume the initial value of position errors along the *Y* axis is 0.5 m, and the initial value of yaw angle error is 0.2 rad.

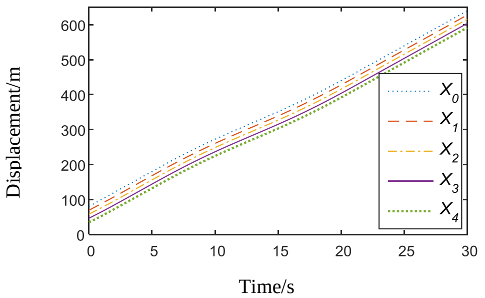
Simulation results of vehicle following and trajectory tracking for lane changing are given in Figures 2 to 4. Figure 2 (a), (b), (c) and (d) show the acceleration, velocity, displacement and inter-vehicle spacing error of the guiding vehicle and four following vehicles along the *X* axis, respectively. Figure 3 (a), (b) and (c) show the actual and expected values of the speed, displacement and tracking error of the vehicle along the *Y* axis during the lane change process of the vehicle M_3. Figure 4(a), (b) and (c) respectively show the yaw rate, yaw angle actual value, expected value and tracking error of the vehicle M_3 when changing lanes. From the figures, the positon tracking error along the *X* axis and the *Y* axis, and the yaw angle error can tend to zero simultaneously with the control method proposed in this paper, it indicates that the control system has asymptotic stability.



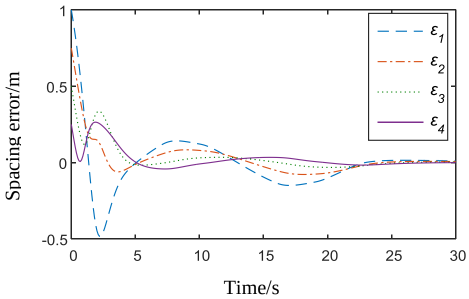
(a) Acceleration along X axis



(b) Velocity along X axis

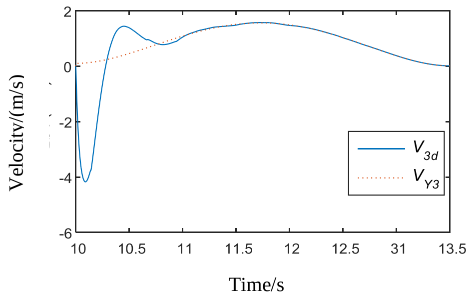


(c) Displacement along X axis

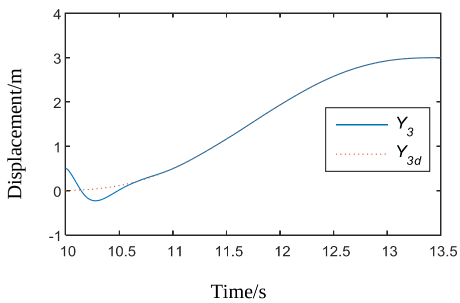


(d) Inter-vehicle spacing error along X axis

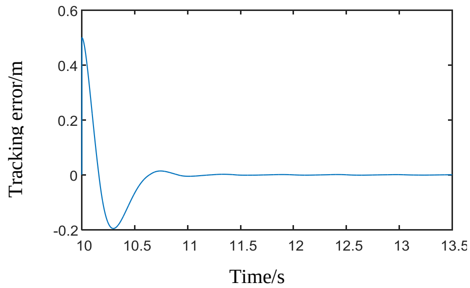
Fig.2: Movement state of the vehicle along X axis



(a) Velocity of the vehicle along Y axis

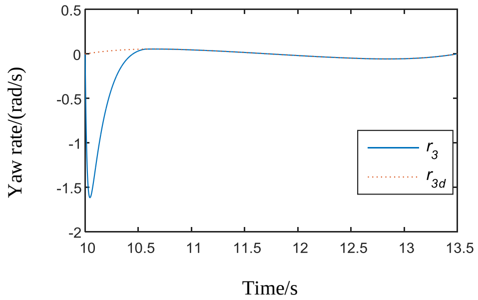


(b) Displacement of the vehicle along Y axis

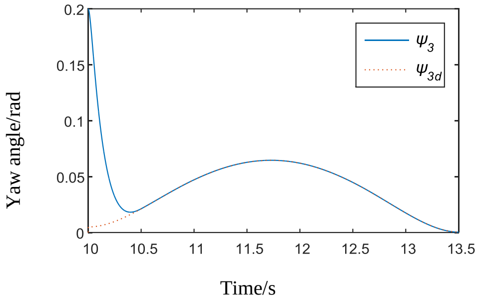


(c) Tracking error of the vehicle along the Y axis

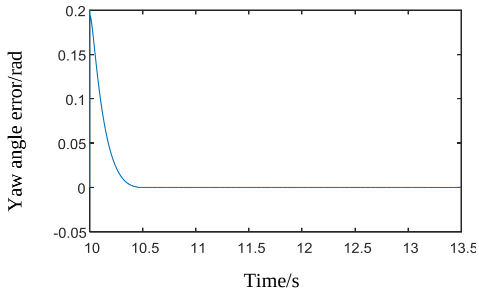
Fig. 3: Movement state of vehicle M_3 along Y axis



(a) Yaw rate of the vehicle



(b) Yaw angle of the vehicle



(c) Yaw angle error of the vehicle

Fig.4: Yaw state of the vehicle

V. Conclusion

In this paper, we study the longitudinal and lateral coordinated control for vehicle following and lane change of vehicle in a platoon. The trajectory model for lane changing is based on odd-order multinomial. The control law is designed by sliding mode variable structure method. We use Lyapunov function to analyze the stability of the control system. Due to the dynamic model of the front and rear wheels is used, the control law designed in this paper can ensure that the position and yaw angle error asymptotically approaches zero and the longitudinal speed is not assumed to be constant. The vehicle maintains following the preceding vehicle in the longitudinal direction while achieving the lane change behavior in the lateral direction, compared with the existing trajectory tracking control method for lane change, the control law is more general.

Acknowledgements

This work was financially supported by the Shandong Provincial Natural Science Foundation, China(ZR2015FM024).

References

[1]. Scarinci R, Hegyi A, Heydecker B G. Definition of a merging assistant strategy using intelligent vehicles. Transportation Research Part C: Emerging Technologies, 2017, 82: 161-179.
[2]. Barmpounakis E N, Vlahogianni E I, Golias J C. Intelligent transportation systems and powered two wheelers traffic. IEEE Transactions on Intelligent Transportation Systems, 2016,17(4): 908-916.
[3]. Chaturvedi M, Srivastava S. Multi-modal design of an intelligent transportation system. IEEE Transactions on Intelligent Transportation Systems 2017, 18(8): 2017- 2027.
[4]. Xuan J Y, Lu G, Wang J C. Vehicle Following Control System Based on Micro Smart Car. Information and Control, 2014,43(2): 165-170.

[5]. Guo G, Yue W. Hierarchical platoon control with heterogeneous information feedback. IET Control Theory and Applications, 2011,15(5): 1766- 1781.
[6]. Ren D B, Zhang J M, Zhang J Y. Sliding mode control for vehicle following with parametric uncertainty. Electric Machines and Control, 2010,14(1): 73-77.
[7]. Guo Lie, Ge Pingshu, Yue Ming, et al. Lane changing trajectory planning and tracking controller design for intelligent vehicle running on curved road. Mathematical problems in engineering, 2014, 3(6):1-9.
[8]. Zheng Z D. Recent developments and research needs in modeling lane changing. Transportation Research Part B, 2014, 60:16-32.
[9]. Rahman M, Chowdhury M, Xie YC, et al. Review of microscopic lane changing models and future research opportunities. IEEE Transactions on Intelligent Transportation Systems, 2013, 14(4):1942-1956.
[10]. Schubert R, Schulze K, Wanielik G. Situation assessment for automatic lane change maneuvers. IEEE Transactions on Intelligent Transportation Systems, 2010, 11(3): 607-616.
[11]. Nilsson J, Brännström M, Coelingh E, et al. Lane change maneuvers for automated vehicles. IEEE Transactions on Intelligent Transportation Systems, 2017,18(5):1087-7096.
[12]. Li Wei, Gao Dezhi, Duan Jianmin. Research on lane change model for intelligent vehicles. Journal of Highway and Transportation R esearch and Development,2010,27(2): 119- 123.
[13]. Ren Dianbo, Zhang Jingming, Cui Shengmin. Longitudinal and lateral coupling control for lane change. Journal of Traffic and Transportation Engineering, 2009,9(3):112-116.
[14]. Ren Dianbo, Cui Shengmin, Wu Hangzhe. Preview control for lane keeping and its steady state error analysis. Automotive engineering, 2016,38(2): 192-199.
[15]. Zhang Guanzhe, Ren Dianbo, Cui Shengmin. Lane changing control of four wheels steering vehicle in intelligent transportation system. Journal of Dalian Maritime University,2011, 37(3):81-84.
[16]. Xiang Yong, Luo Yugong, Cao Kun, et al. Automatic Lane change control based on vehicle to vehicle communications. Journal of Highway and Transportation Research and Development, 2016, 33(3): 120-126, 145 .
[17]. Yang D, Zhu L L, Ran B, et al. Modeling and analysis of the lane changing execution in longitudinal direction. IEEE Transactions on Intelligent Transportation Systems, 2016,17(10):2984-2992.
[18]. Fang Z P, Duan J M, Zhang B G. Cascade control for stable lane keeping by intelligent four-wheel steering vehicles. Information and Control, 2016, 45 (4): 492-498.
[19]. Yang M D, SONG J C, Yang P P, et al. Consensus based platoon for autonomous vehicles with time varying inter-vehicle distance.Control and Decision, 2017,32(12):2296-2300.
[20]. Li Wei, Wang Jing, Duan Jianmin. Lane changing trajectory planning of intelligent vehicles based on polynomials. Computer Engineering and Applications, 2012, 48(3):42-245.

Dianbo Ren1 “Cooperative Control for Longitudinal Following and Lateral Lane Changing of Intelligent Cars." IOSR Journal of Electrical and Electronics Engineering (IOSR-JEEE) 13.4 (2018): 12-21.